18

Solutions

In-Class Activities

Activity 18-1: Generation M

a. This is a statistic because it is a number that represents a sample.

b. For a 99% confidence interval, you calculate .68 ± (2.576)(.010348) = .68 ± (.02666) = (.6533, .7066).

c. The values .70 and .6667 are in this interval; .65 and .707 are not.

d. A significance test should reject the hypothesis that $\pi = .65$, but .7 is contained in the interval, so this could be a plausible value for π. A significance test would not reject the hypothesis that $\pi = .7$.

e. Using Minitab's Test and CI for One Proportion:

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Test of p = 0.65 vs p not = 0.65
Sample     X     N  Sample p        99% CI          Z-Value  P-Value
1       1382  2032  0.680118  (0.653465, 0.706771)     2.85    0.004
Using the normal approximation.
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f. See table following part g.

g. Here is the completed table:

Hypothesized Value	Contained in 99% Confidence Interval?	Test Statistic	*p*-value	Significant at .01 Level?
.65	No	2.85	.004	Yes
.6667	Yes	1.28	.199	No
.7	Yes	−1.96	.05	No
.707	No	−2.66	.008	Yes (barely)

h. If a hypothesized value is contained in the 99% confidence interval, then this value is *not* significant at the .01 level, and vice versa.

Activity 18-2: Pet Ownership

a. Because this number (.316) describes a sample, it is a statistic, represented by $\hat{p}$.

b. Let π represent the proportion of all American households who own a pet cat.

The null hypothesis is that one-third of all American households own a pet cat. In symbols, the null hypothesis is H_0: $\pi = .333$.

The alternative hypothesis is that the proportion of American households who own a pet cat differs from one-third. In symbols, the alternative hypothesis is H_a: $\pi \neq .333$.

The test statistic is $z = \dfrac{.316 - .333}{\sqrt{\dfrac{(.333)(.667)}{80{,}000}}} = -10.20$.

Using Table II, p-value $= 2 \times \Pr(Z < -10.20) < .0002$.

Reject H_0 with this very small p-value.

You have overwhelming statistical evidence that the proportion of all American households who own a cat differs from one-third.

c. For a 99.9% CI, you calculate $.316 \pm (3.291)\sqrt{.316(1 - .316)/80000} =$ $.316 \pm (3.291)(.001644) = (.310591, .321409)$. You are 99.9% confident the proportion of all American households who own a pet cat is between .311 and .321.

d. Yes, this confidence interval is consistent with the test results because $1/3 \approx .333$ is not contained in the interval.

e. Yes, the sample data provide *very strong* evidence that the population proportion (π) is not one-third. The p-value is what helps you decide this; the p-value is so small (essentially zero) that it easily convinces you that π is not one-third.

f. No, the sample data do not provide strong evidence that the population proportion of households who own a pet cat is very different from one-third. The evidence suggests that this proportion is between .311 and .321, which are awfully close to .33. The confidence interval helps you decide how much π differs from one-third.

Activity 18-3: Racquet Spinning

a. Sample proportion: .565 Test statistic: 1.84 *p*-value: .066 Significant at .05? no

b. Sample proportion: .575 Test statistic: 2.12 *p*-value: .034 Significant at .05? yes

c. Sample proportion: .65 Test statistic: 4.24 *p*-value: .000 Significant at .05? yes

d. The sample results are most similar in parts a and b, where you had almost the same number of "ups."

e. The decisions are the same in parts b and c (where the sample results are quite dissimilar).

Activity 18-4: Female Senators

a. For a 95% CI, you calculate $.16 \pm 1.96(.0367) = (.088, .231)$.

b. No, this confidence interval is not a reasonable estimate of the actual proportion of all humans who are female.

c. The confidence interval procedure fails in this case because the alien did not select a simple random sample of all humans. The U.S. Senate is not representative of the population of all humans with respect to gender, so the sampling method is extremely biased and you cannot legitimately use the confidence interval procedure.

d. You do not need to estimate the proportion of women in the 2007 U.S. Senate. You know this proportion is .16.

Activity 18-5: Hypothetical Baseball Improvements

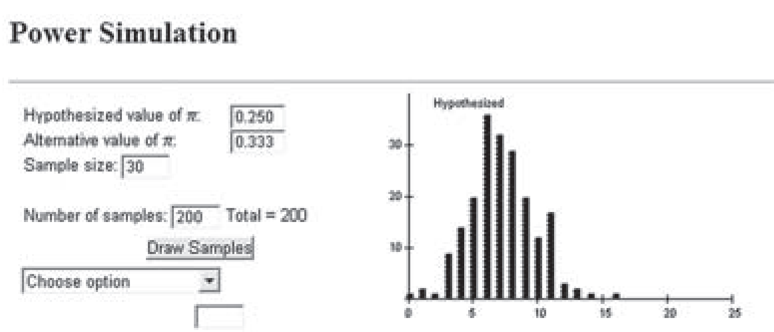
a. The null hypothesis is that this player is still a .250 hitter. In symbols, H_0: $\pi = .250$.

The alternative hypothesis is that this player has improved and is now better than a .250 hitter. In symbols, H_a: $\pi > .250$.

b. A Type I error would be deciding that the player has improved his batting performance when, in fact, he is still batting no better than .250.

c. A Type II error would be failing to realize that the player has improved.

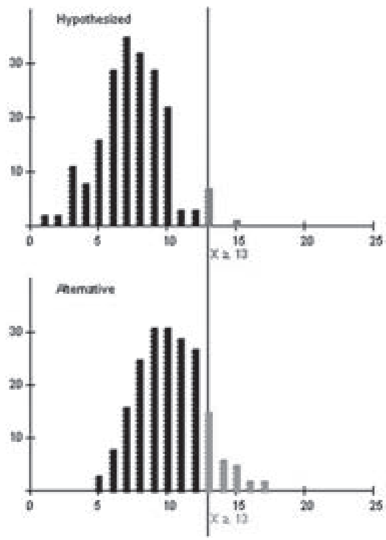
d. Answers will vary. The following is a representative set:



This distribution is roughly normal, centered at about 7.5 hits, and extends from about 1 hit to about 17 hits.

e. A player would need to get at least 13 hits.

f. There is a great deal of overlap between the two distributions.

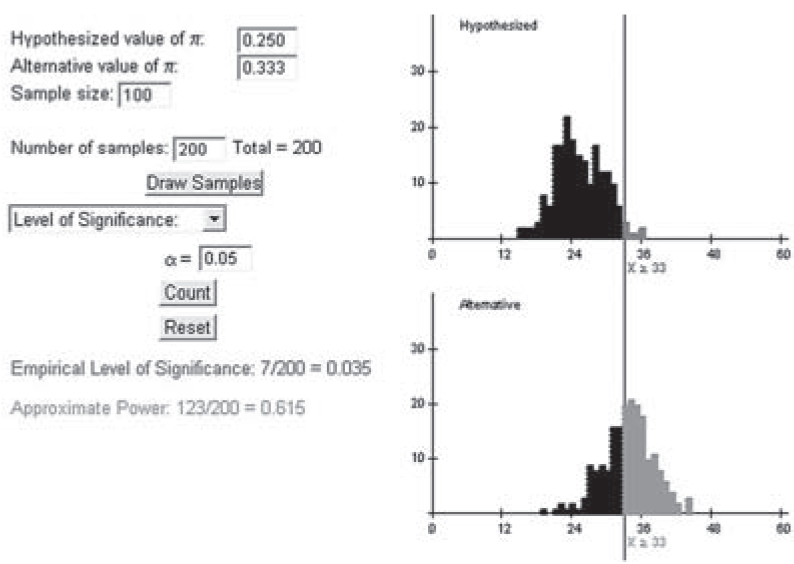


g. 28/200 = 14%

h. No, it does not appear very likely that a .333 hitter will be able to establish that he is better than a .250 hitter in 30 at-bats. Based on this simulation, he had only about a 14% chance of establishing his improvement (performing well enough to convince the manager that his success rate was now greater than .250).

i. Power ≈ .14

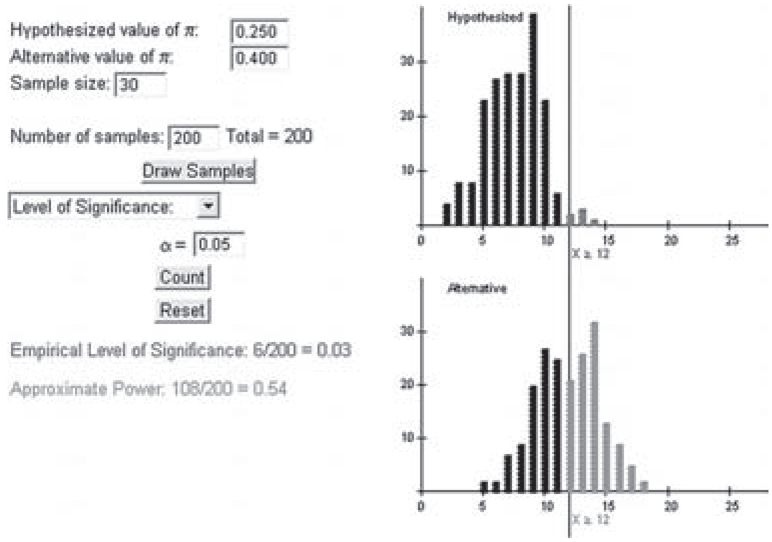
j. The following is based on one representative running of the applet:



Based on this simulation, a player would need at least 33 hits (out of 100) in order for the probability of a .250 hitter to do that well by chance alone to be less than .05. The approximate power of this test is 123/200 or .615.

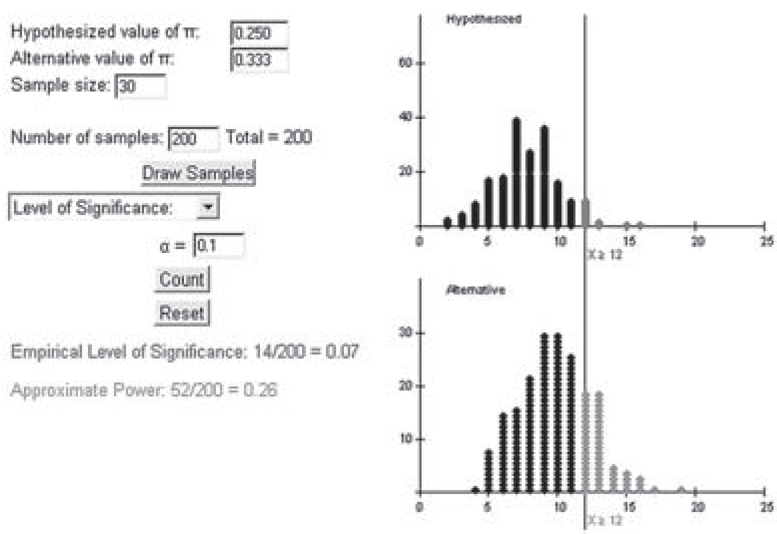
k. Answers will vary by student expectation, but the test will be more powerful if the player improves to a .400 hitter. It should be easier to detect the improvement to .400 because it is farther away from .250 than .333 is.

The simulation confirms this result; the approximate power is 108/200 or .54 (using 30 at-bats).



l. Answers will vary by student expectation, but the test will be more powerful if you use a higher significance level.

The simulation confirms this result; the approximate power is 52/200 or .26 (using 30 at-bats, alternative value of $\pi = .333$ and the significance level .10).



m. i. The magnitude of the difference between the hypothesized value π_0 and the particular alternative value of π

ii. The significance level

Activity 18-6: *West Wing* Debate

a. The population of interest is all adult Americans who are familiar with these fictional candidates. The parameter (call it π) is the proportion of this population who would have supported Santos if they had been asked.

b. The 90% CI for π is $.54 \pm .024$, which is (.516, .564).

The 95% CI for π is .54 ± .028, which is (.512, .568).

The 99% CI for π is .54 ± .037, which is (.503, .577).

c. The midpoints are all the same, namely .54, the sample proportion of Santos supporters. The 99% CI is wider than the 95% CI, and the 90% CI is the narrowest.

d. Yes. All three intervals contain only values greater than .5, so they do suggest, even with 99% confidence, that more than half of the population would have favored Santos.

e. H_0: $\pi = .5$ (half of the population favored Santos)

H_a: $\pi > .5$ (more than half of the population favored Santos)

f. Because all three intervals fail to include the value .5, you know that the *p*-value for a two-sided alternative would be less than .10, .05, and .01. Because you have a one-sided alternative in this case, you know that the *p*-value will be less than .01 divided by 2, or .005 (because the observed sample proportion is in the conjectured direction).

g. A Type I error occurs when the null hypothesis is really true but is rejected. In this case, a Type I error would mean that you conclude that Santos was favored by more than half of the population when in truth he was not favored by more than half. In other words, committing a Type I error means concluding that Santos was ahead (favored by more than half) when he wasn't really. A Type II error occurs when the null hypothesis is not really true but is not rejected (you continue to believe a false null hypothesis). In this case, a Type II error means that you conclude Santos was not favored by more than half of the population when in truth he was favored by more than half of the population. In other words, committing a Type II error means concluding that Santos was not ahead when he really was.

h. The test would be more powerful if Santos really were favored by 55% rather than 52%. The higher population proportion would make it more likely to reject the null hypothesis that only half of the population favored Santos because the distribution of sample proportions would center around .55 rather than .52 (further from .5).

i. The larger sample (10,000) would produce stronger evidence that more than half of the population favored Santos. With less variability in the sampling distribution, the *p*-value would be much smaller.